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ME570 HW2

Professor Tron

5 October 2020

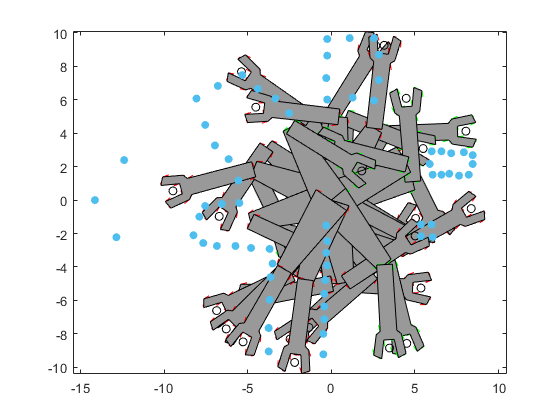
*Q1.1*:

1. R1(θ) represents a 2-D CCW rotation on the x2 x­3 plane.
2. R2(θ) represents a 2-D CCW rotation on the x1 x­3 plane.
3. R3(θ) represents a 2-D CCW rotation on the x1 x­2 plane.
4. R4(θ) represents a 2-D CCW rotation on the x1 x­2 plane, which is then negated (in other words, an additional π rad CCW atop the theta input).

*Q2.1*:

1. and

*Q2.2*:



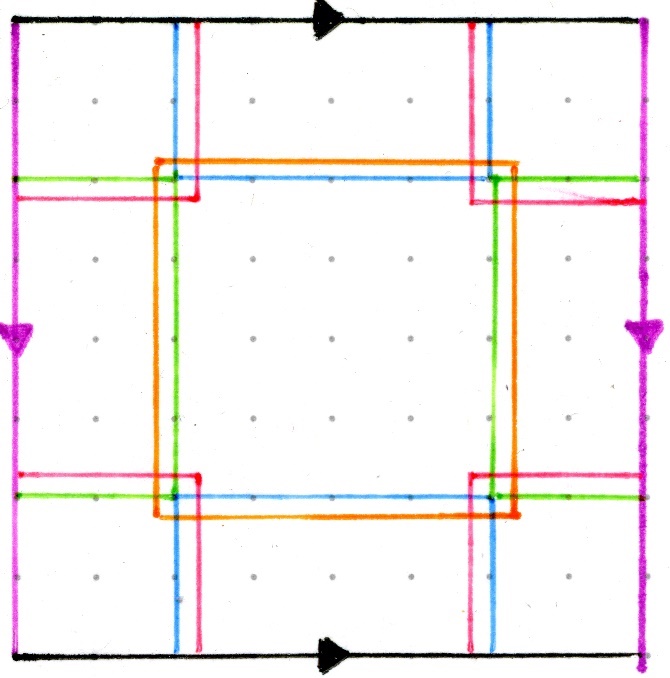
*Q4.1*:

(Also demonstrable by comparing to mapping between complex numbers and angle, I believe).

Show:

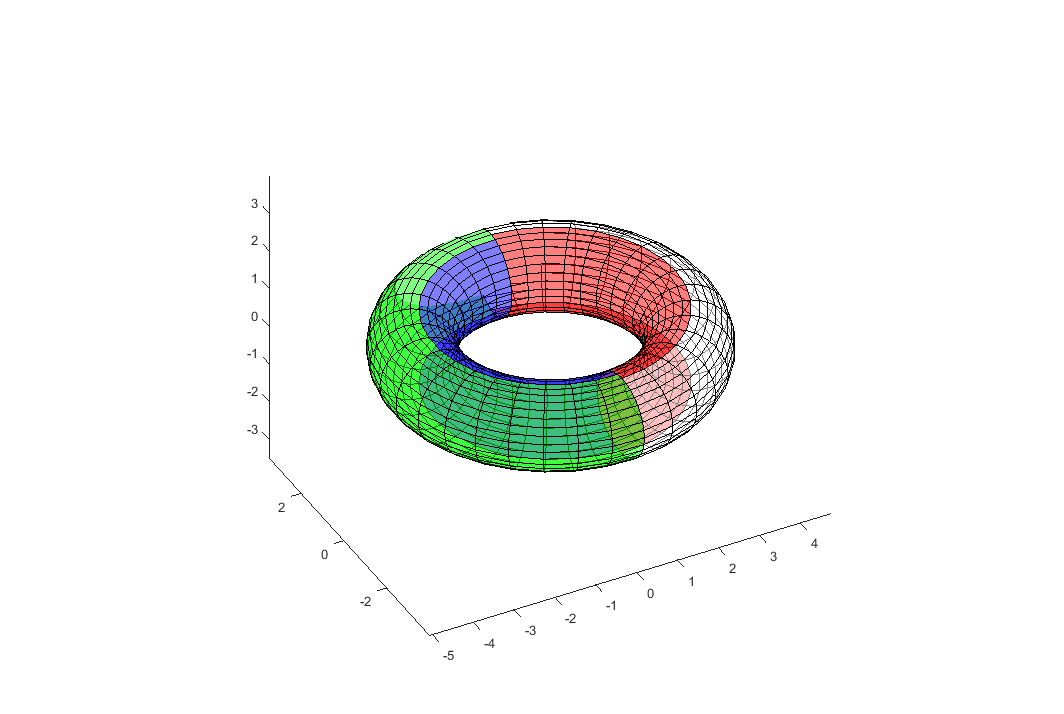
*Q5.1*:

As , and requires 2 charts, it follows easily that for , 4 is sufficient. Pictured on a flat torus and with the restriction of only using square regions within , it becomes evident that 4 is necessary as well:



*Q5.2*:

The same charts, as applied to the surface of the torus in 3D, with the colors green, blue, red, and white, respectively.

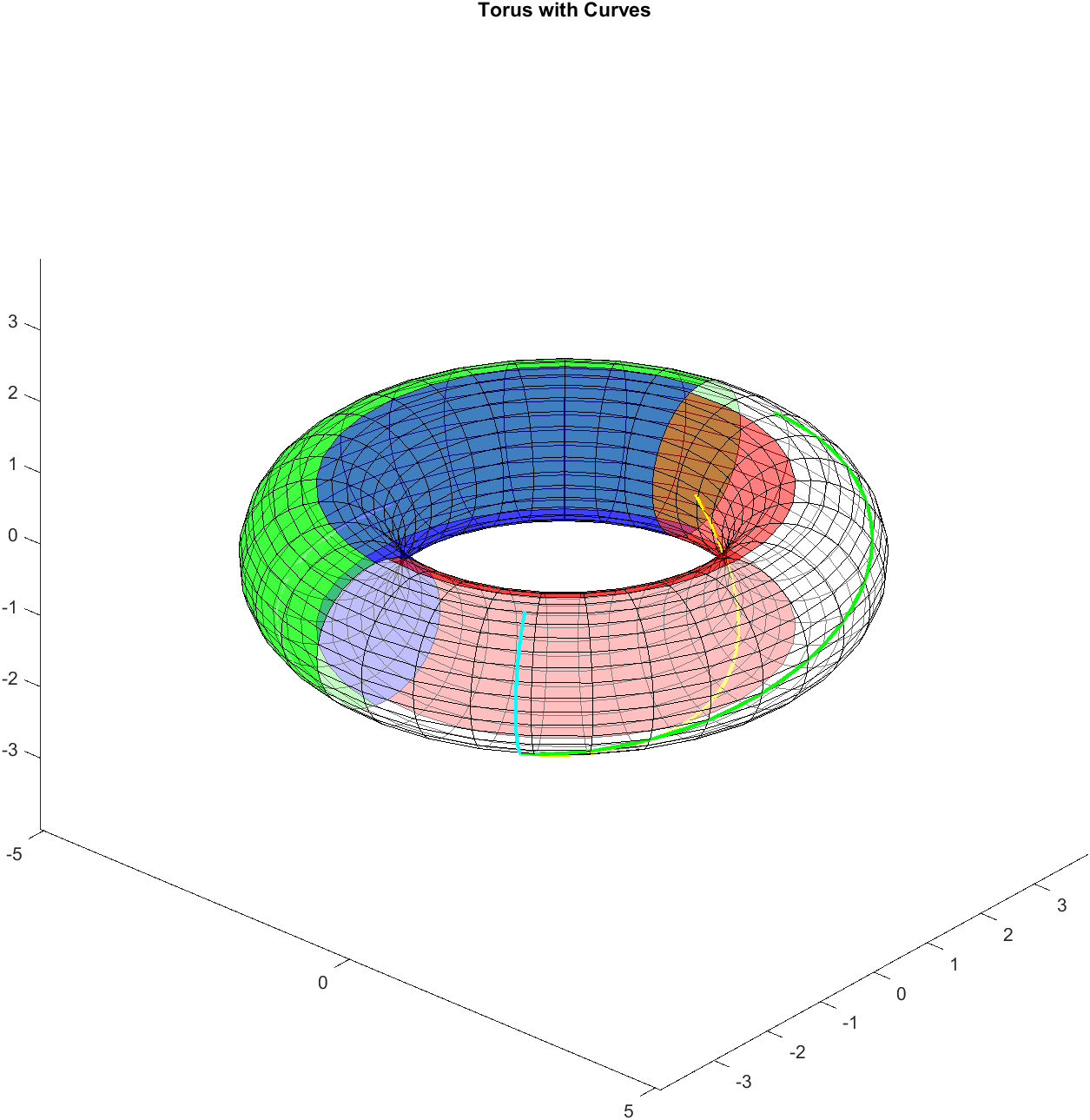


*Q5.3*:

If charts self-overlapped, the mapping would no longer be diffeomorphic; the same point on the surface could be represented multiple ways on a single chart, defeating the chart’s purpose. If sections of the torus were uncovered, one would not have a full atlas of the topology; there would exist points in the space that one could not map to, again defeating the purpose.

*Q5.4*:

*Q5.5*: Torus with curves.



*Q6.1*:

From Q2.1,

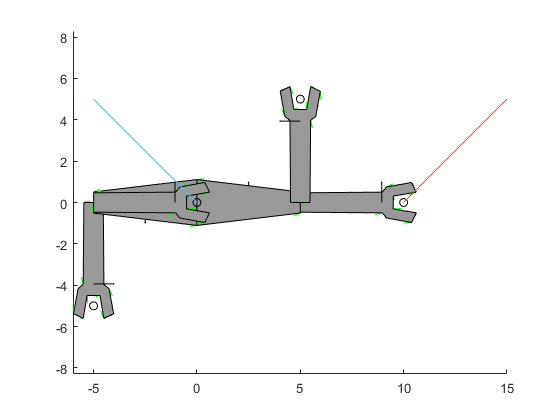
Thereby, . As both and are functions of , to find , we calculate:

*Q6.2*:

Utilizing the program written for *code Q6.1*, with the equation detailed in *report 6.1*, with the inputs:

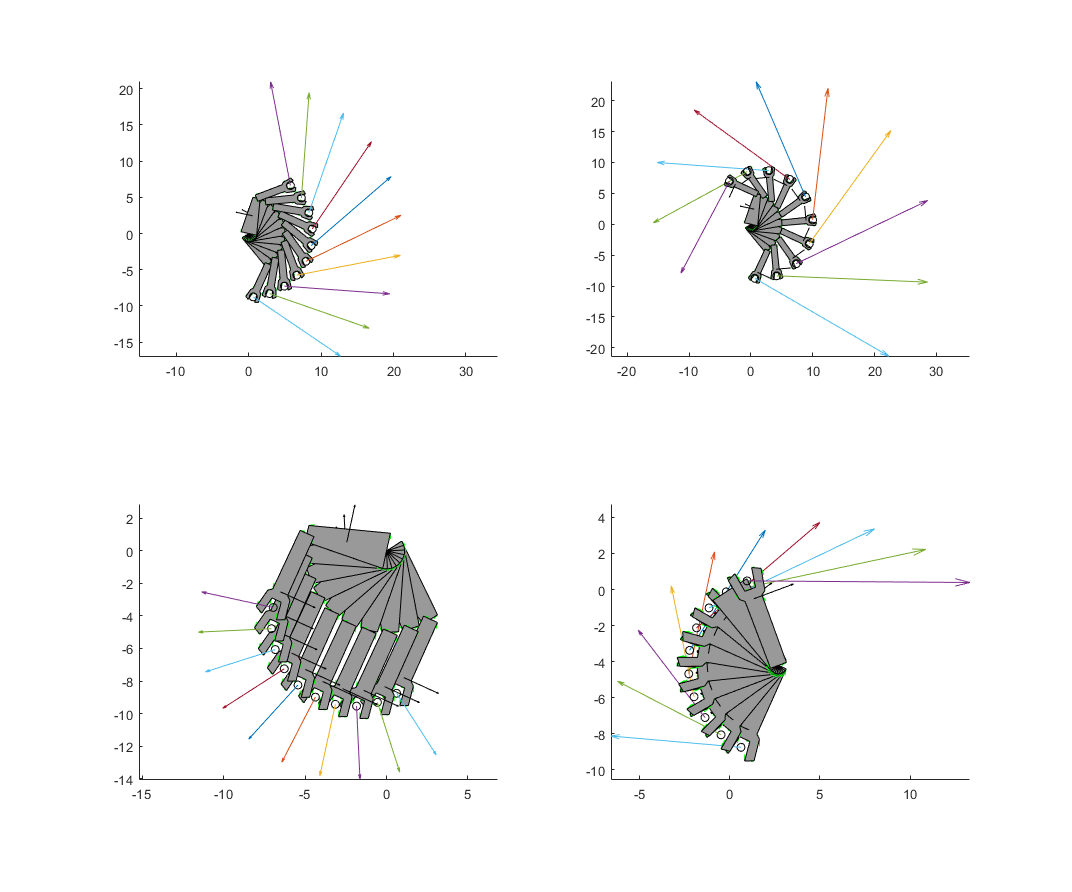
We get the result:

Using twolink\_plot.m, with the states superimposed, we get:



Likely the oddest feature of the tangents is how they remain the same when a(1) and a(2) are switched from [1;0] to [0;1].

*Q6.3*:



*Q6.4*:

The arms traversing the XY plane corresponds to lines travelling along the surface of the torus in *provided 5.2*, mapped via the two arm angles. As such, the tangents are interrelated between the two spaces.

*Q7.1*:

This homework took approximately 12 hours for me to complete, the majority of that attributable to this being my first homework for this class (late join), MATLAB brush-up work required (far greater familiarity with other languages such as Python–the last time I used MATLAB intensively was 3ish years ago), and a steep personal learning curve for some of the linear algebra, due to a less intensive background in it. Altogether, once done, the concepts and work make sense and appear fairly trivial, syntax was truly the sticking point.